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Volume 1, Module 4

Scale and Area Measurement

Scale

Scale is the ratio of a distance on an aerial photograph to that same distance on the ground in the real world. It can be expressed in unit equivalents like 1 inch = 1,000 feet (or 12,000 inches) or as a dimensionless representative fraction (1/12,000) or as a dimensionless ratio (1:12,000).

This is an extremely important concept to internalize; scale determines what you can see, the accuracy of your estimates and how certain features will appear. With experience you will understand the differences between 1:5,000 aerial photography and 1:10,000 scale photography. When conducting an analysis that is based on air photos it will sometimes be necessary to make estimates regarding the numbers of objects, the area covered by a certain amount of material or it may be possible to identify certain features based on their length. To use this dimension of air photo interpretation it will be necessary for you to be able to make estimates of lengths and areas which requires knowing the photo's scale. Sometimes this is printed on the photo, but you should never trust it, and sometimes it is unknown. This module is designed to give you the basics necessary to determine photo scale and make estimates of length and area.

The diagram below illustrates some important concepts about the geometry of the flat surface that apply to the calculation of scale and area from air photos. The first thing to notice is that the distance from D to E and A to B are proportional to the ratio of the focal length (f) to the height above the ground (H). This allows for the calculation of proportional lengths because the angles formed on either side of the lens, labeled point C on the diagram, are identical. Also note that the center point of the image (the Principal Point, PP) and the actual center point on the ground (Nadir) fall along the optical axis of the camera in this idealized diagram.

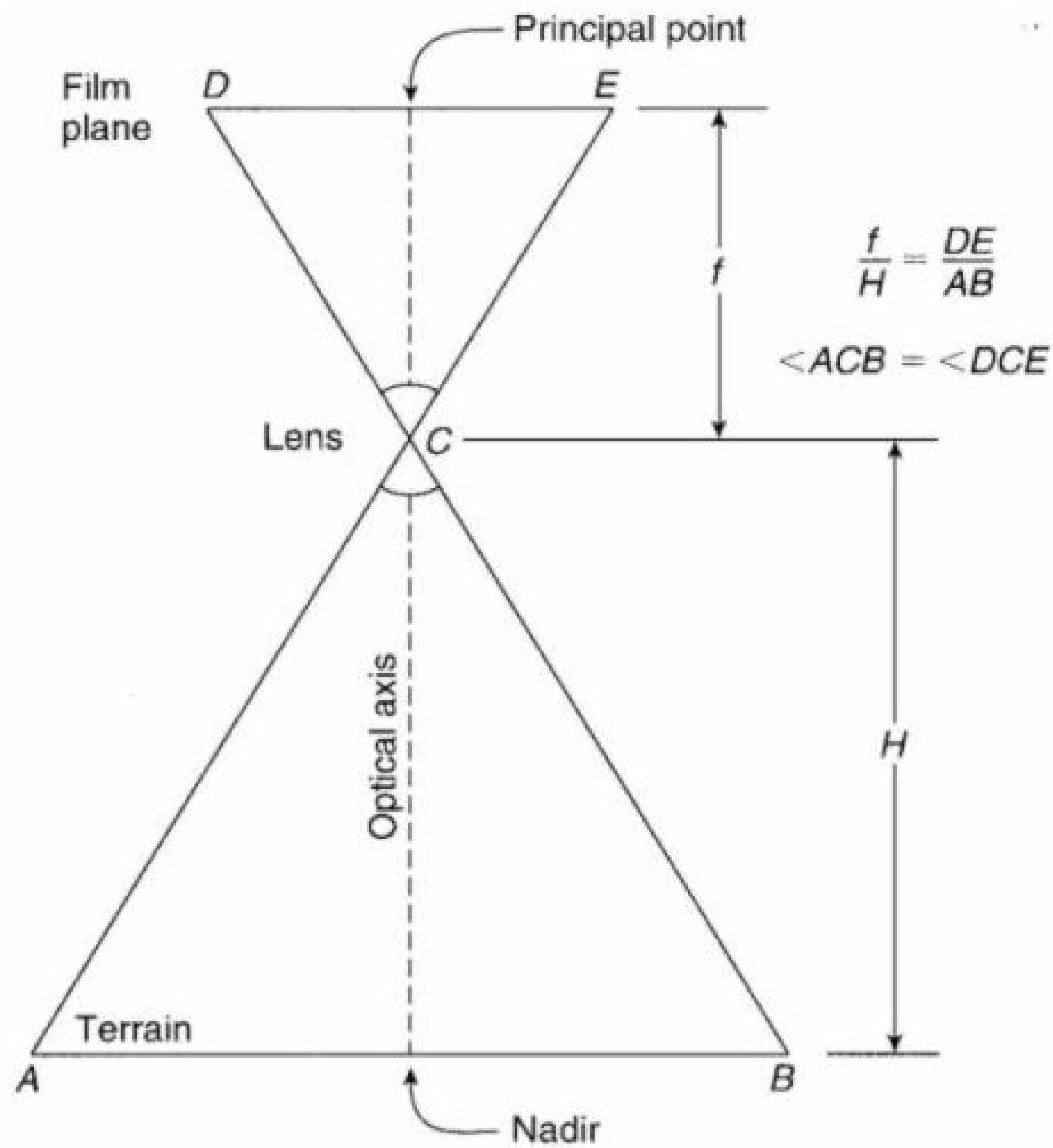
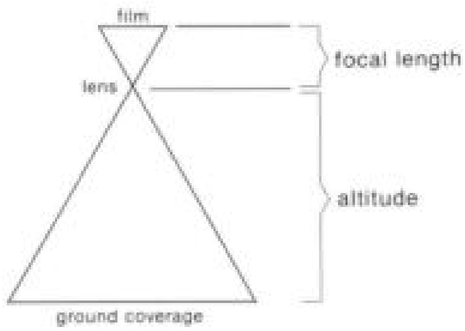


Photo Scale

Knowledge of the camera focal length and the aircraft altitude makes it possible to determine photo scale (PS) and the representative fraction (RF) of a photo.



The photo scale and representative fraction may be calculated as follows:

PS = f / H

Variables: PS - Photo Scale, f - camera focal length, H - altitude above the ground

*Photo Scale is equal to camera focal length divided by the **H**eight (altitude) of the plane.*

RF = 1 / (H / f)

Variables: RF - Representative Fraction, f - camera focal length, H - altitude above the ground *Representative Fraction (RF) is equal to one divided by the ratio of altitude (**H**) and camera **f**ocal length.*

While the foregoing method of deriving photo scale is theoretically sound, it often happens that either camera focal length or altitude above the ground are unknown. In such cases, scales may be determined by the ratio of photo distance between two points to map distance (MD) using the map scale (MS) or ground distance (GD) between the two points.

PS = PD / GD

Variables: PS - Photo Scale, PD - Photo Distance, GD - Ground Distance

PD and GD are different due to the source the measurement is referring to. Ground Distance (GD) and Map Distance (MD) are used to differentiate a measurement you make from the map source and a real world distance that you calculate from map scale, measure using the map's scale bar or measure yourself with a measuring tape in the field. When calculating scale, PD (Photo Distance) and GD (the real world Ground Distance) must be in the same units in order to yield a unitless Representative Fraction (RF); the Map Scale Reciprocal (MSR) and the Photo Scale Reciprocal (PSR) are both unitless.

RF = 1 / [(MD*MSR)/PD)] or **RF = 1 / [(PD*PSR)/MD)]**

Variables: MD = Map Distance, MSR = Map Scale Reciprocal, PSR - Photo Scale Reciprocal, PD - Photo Distance

In applying this technique, the two points selected should be diametrically opposed in such a way that a line connecting them passes near the principal point (PP). If the points are approximately equidistant from the PP, the effect of photographic tilt upon the scale measurement will be minimized. Features selected must also be chosen for easy recognition and measurement. Flat terrain is preferred; hilly terrain should be avoided to minimize the effects of relief displacement. The significance of the principle point and the nadir, as well as relief displacement, will be discussed later with regard to the geometry of air photos. For the concepts introduced in this module it is necessary to focus on the basics.

Ground distance can be measured with surveying equipment, it may be known in advance, it can be calculated by multiplying the measured distance on a map by the map scale, or it can be approximated using the map's scale bar. If the path or road you wish to measure curves you can use a piece of string or a ruler to measure the length; lay the piece of string on the road segment, and then straighten it out and measure how long it is.

Look at the portion of the USGS Quadrangle below. The scale bar at the bottom can be used to measure a map distance (MD). The corresponding distance on the photo you measure with a ruler. Your MD and PD measurements need to be in the same units (m, ft, in or cm) for the calculations.

The part of the scanned map that shows our area of interest has been rescaled slightly from the original source map. Use the scale bar and the edge of a piece of paper to measure off the lengths of road segments you can locate on the map and the photo. Using the scale bar segments labeled with feet, measure the distance of a road, this will be MD. Now find that same road segment on the photo and measure it with a ruler in inches, this will be PD. Remember that MD and PD have to be in the same units, so you have to convert one of the measurements before doing the calculation so the units cancel or the resulting fraction is unitless.

UCSB - IV in 1954


UCSB - IV Goleta USGS Quad

Expressing Scale

1. Scale ratio is also referred to as the proportional scale. 1:20,000 is read as "one to twenty thousand". The scale ratio is always written as one unit on the photo or map to the corresponding number of units on the ground.
2. Representative fraction scale (RF): Two other terms refer to the representative fraction scale - the fractional scale and the RF scale which is the scale ratio written in fractional form, 1/20,000.
3. Equivalent scale: Equivalent scale is also known as the descriptive scale. For example: one inch equals 5,280 feet (1 inch = 5,280 feet); two inches equals one mile (2 inches = 1 mile); and 100 feet per inch.
4. Graphic scale: Also called a bar scale, used on maps and drawings to represent length scale on paper with length units.

Representative Fractions and Equivalent Scales

This chart lists some common representative fractions and the equivalent scales in words. The unit conversions involved are important to be able to calculate yourself, this chart provides some examples.

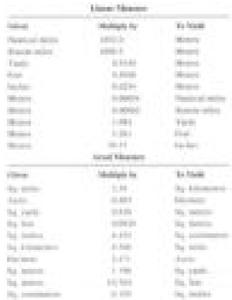
Conversions

Distance Unit conversions are important to understand, it is not necessary to memorize all of them but you should be aware of the more common ones. Also, it is important to get a grasp of the magnitude of some units relative to other units; if you're converting from feet to meters, should the resulting number be bigger or smaller? If you are not familiar with unit conversions, or you are calculator dependent, you may need some practice setting up the calculations so that the units cancel.

Common units and transformations

1 meter = 100 centimeters = 1,000 millimeters
1 foot = 12 inches, 1 yard = 3 feet, 1 meter = 3.28 feet

1 square meter = 10.76 square feet , 1 acre = 43,560 square feet,
1 square kilometer = 230.4 acres



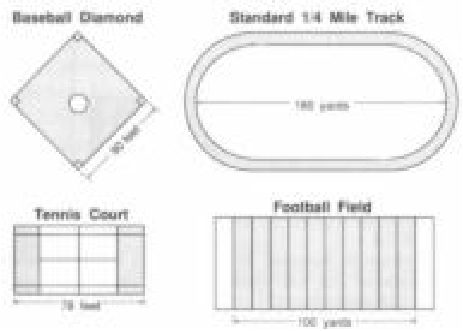
If one inch on the photo is equivalent to 1,000 feet on the ground (or 12,000 inches) : RF = 1/12,000; MS = 1/12,000; MSR = 12,000 and the map scale denominator is also equal to 12,000, *different words for the same thing!* The 12,000 part is what is important, that is the real world distance per unit distance on the photo or map, it doesn't have to be inches, it can be millimeters or centimeters or whatever. The important part to remember is that both terms are inches, or whatever unit you choose, so the resulting fraction is unitless.

Area Measurements

The important thing to keep in mind, once you have mastered measuring distances, is that areas have squared units. For a rectangular area its length x width, so if you measure both and convert these distances remember that if you are multiplying them together the resulting units are squared.

For example, if an area is 100 meters by 500 meters, it is 50,000 square meters. Now if you wanted to change that number to square feet its not x 3.28, its x 10.76 (3.28x3.28), there are 3.28 feet per meter. Also, it helps to think it though; if you're converting from square meters to square feet should the resulting number be bigger or smaller? Knowing units and distances will help you learn how to arrange the calculations and to recognize incorrect calculations.

The precision of a measurement is dependent upon; your ability to determine photo scale, the precision of the conversion factors, the precision of the measuring device (e.g. using a standard 12 inch ruler or a millimeter scaled ruler) and the accuracy with which you can determine the edges of a feature or area.



Athletic fields have standard dimensions, you can use these lengths to calculate photo scale.

Techniques for area measurements:

1) Polar Planimeters

2) Transect

Line intercept or transect method of canopy estimation is analogous to the dot grid method and is similarly accurate. In this method lines are superimposed on the aerial image and the length of each line that overlays tree canopy is compared to the total line length. Canopy cover is then calculated as: % canopy cover = 100 x (length of lines covered by trees/total length of lines in sample)

Lines may be printed on a transparent sheet or can be designated by randomly dropping a clear scale on the photo. If streets or other features are arranged in parallel lines, sampling bias is best avoided by using a random arrangement of lines rather than parallel lines on the sampling overlay. Accuracy is improved by using more short lines rather than a few long lines.

3) Dot Grids

Dot grid area estimations involve laying a transparent grid over an area of interest and counting the grid cells or dots that fall within that area. This is a quick and easy way to estimate areas, or to estimate the density of objects, and is relatively easy to understand. Each dot or grid cell is proportional to an area according to the scale of the source image, summing the number of dots or grid cells and multiplying by the scale of conversion allows you to estimate areas quickly.

Dot grid method of canopy estimation

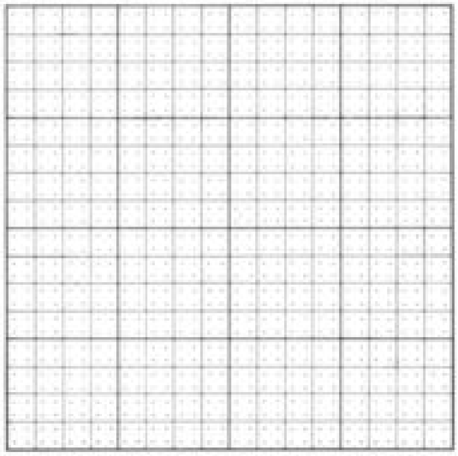
This is an easy, accurate, and relatively rapid method for determining canopy cover, and is equally applicable to natural woodlands and planted urban forests. A dot grid is a sheet of transparent material imprinted with dots arranged in a regular grid. Dot grids can be purchased from forestry suppliers or developed with graphics software and printed onto transparency material. The canopy cover estimate is made by laying the dot grid over the area of the aerial photo to be sampled and counting the number of dots that fall on tree crowns. Percent crown cover can then be calculated as: % canopy cover = 100 x (dots falling on trees/total number of dots within sampled area).

Sampling bias may be a problem if a regular dot grid is superimposed on a photo with features that repeat in a regular pattern, such as rectangular city blocks in which case make sure that the dot grid is always skewed relative to the street grid to minimize sampling bias.

Sample size. How many dots do you need to count? Unfortunately, there is no single answer to this question, but you can calculate the minimum sample size of dots required for a given application if you have some basic information about the population of interest. Several basic principals apply when determining the necessary sample size. The reliability of the canopy cover estimate will increase as the dot density increases, but the increase in statistical power begins to plateau at high sample sizes.

The graphic on the left below is an example of the dot grid overlay method. All of the necessary information is given. Remember that these are area units so they are squared, and that the scale given is a linear scale (not are area scale). On the right is a standard dot grid. Note that if this dot grid is printed that the darker lines are intended to be one inch apart, you will need to resize this graphic if you are going to print it on transparencies.

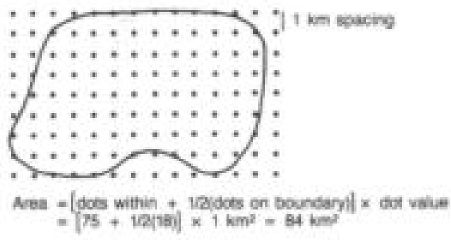
1:20,000, each square contains 25 dots and is 1 cm on a side. How big is this lake?



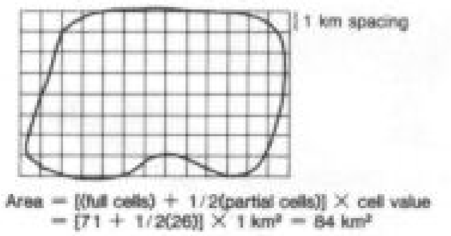
Dot Grid. Scale a grid cell relative to the photo being measured (real world area) and divide by 4 (how much area each dot represents)

The four diagrams below illustrate different methods for estimating area. The scale is fixed for each diagram (1 km). Each of these methods has tradeoffs between precision and accuracy, but all are valid methods of estimating areas.

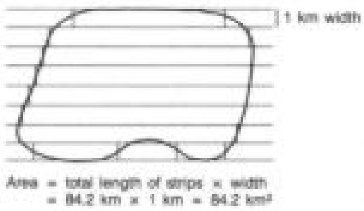
Dot Grid Method



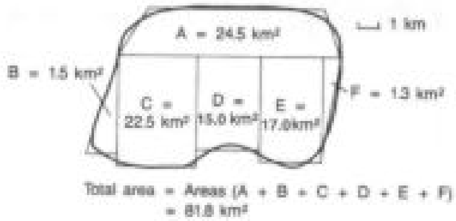
Grid Square Method



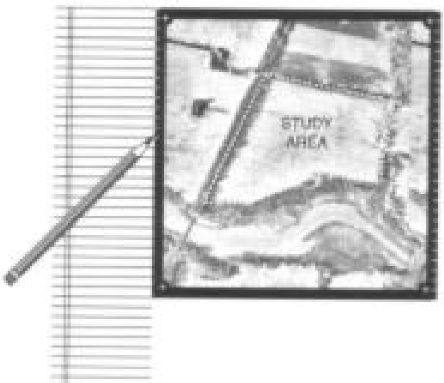
Transect Method



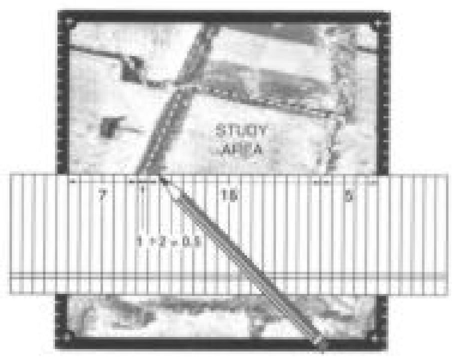
Polygon Method



The two diagrams below illustrate the transect method of estimating area. This method is comparable to the line intercept or transect method described above with regard to tree canopies.



Step 1 - With a piece of lined paper, mark equal spaced lines on the edge of the photo



Step 2 - Count the number of spaces on the notebook paper that falls within the area being measured.

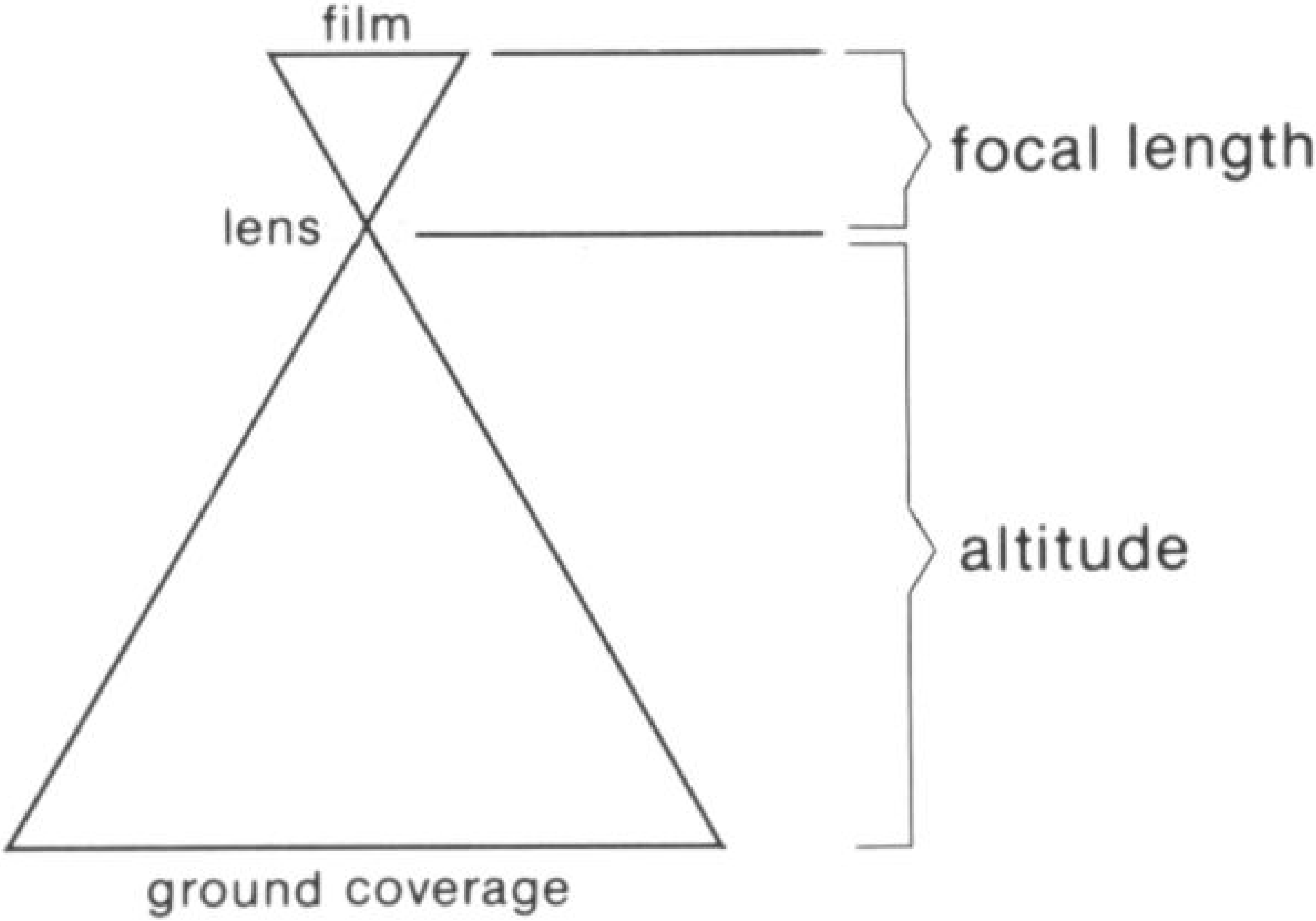
Stereoplotters and GIS

With the techniques described above it will be possible to make estimates of areas and lengths. This can be useful when interpreting air photos because sometimes relative sizes, and differences in areas, can lend support to an interpretation. When exacting measurements are required however dot grids and scaled measurements need an additional level of correction. The devices for performing this correction are called "stereoplotters".

There are two types, analog and digital. Analog stereo plotters require specialized knowledge and calibration but yield extremely accurate measurements when used correctly in conjunction with enough geometric control. The second type of device is called an "analytical stereoplotter" and is digital. The main benefit of these devices is that they are usable by trained individuals and are very reliable when maintained properly.

Geographic Information Systems (GIS) as well as most image processing software packages, have image registration capabilities that have replaced manual area estimation techniques. But not in all cases is GIS feasible or practical, and if you lack sufficient ground control points in order to georectify the imagery the additional cost of doing so may not afford enough accuracy so as to be cost effective.

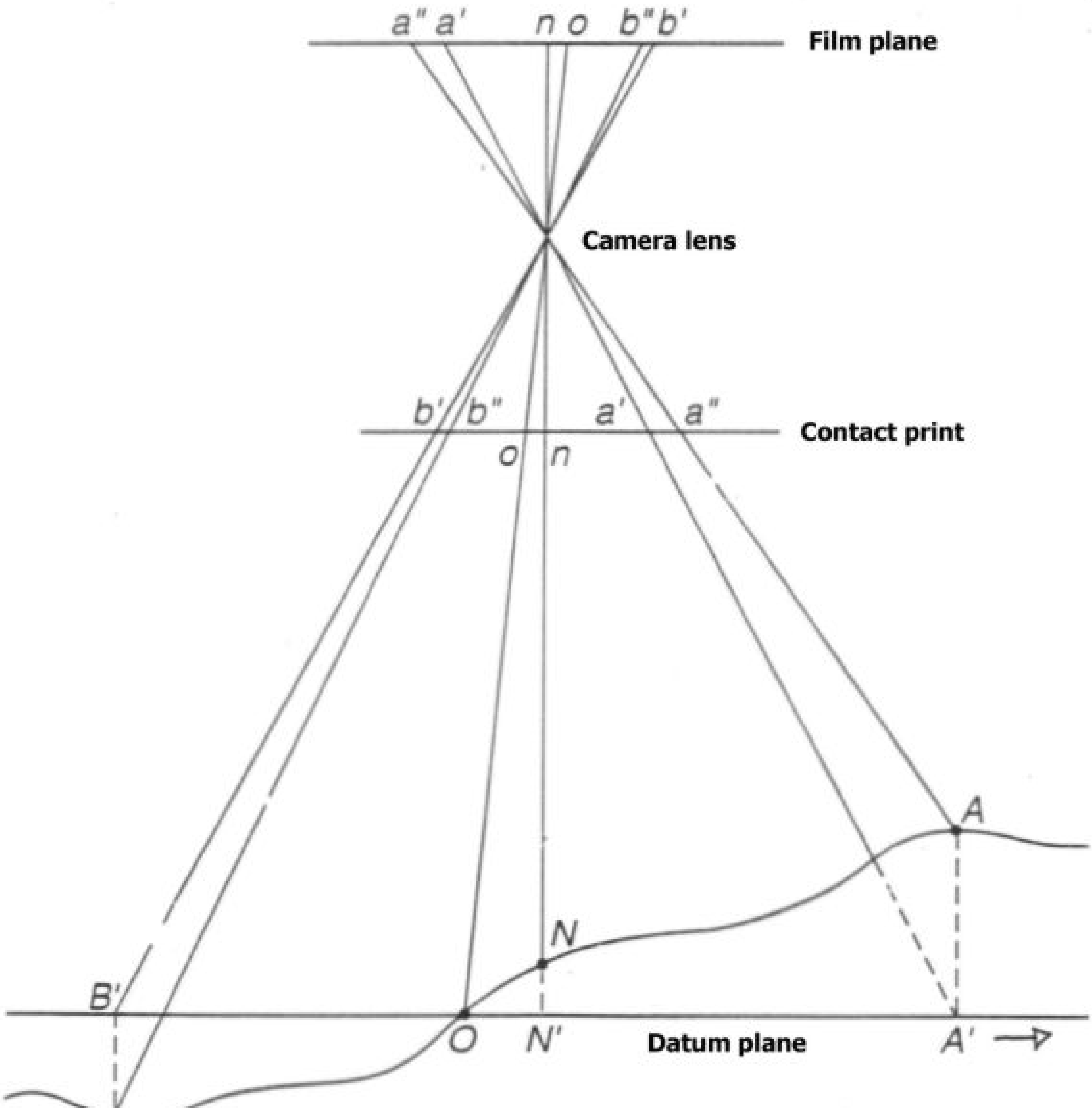
Image registration will be covered in another module, but suffice it to say that the manual techniques discussed so far will provide reasonably accurate inexpensive estimates of area and length.

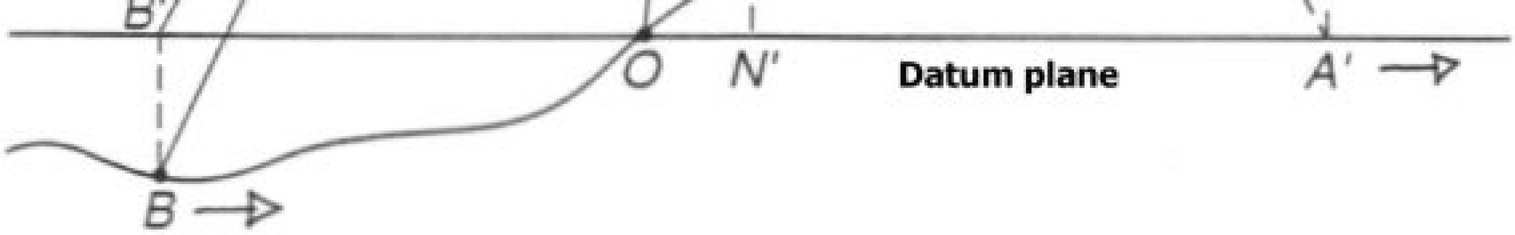

film
focal length
lens
altitude
ground coverage

Relief Displacement

Displacement is the radial distance between where an object appears in an image to where it actually should be according to a planimetric coordinate system. Relief Displacement is caused by changes in the distance between the ground and the camera as the plane flies over the ground. The nadir point is always free of any relief displacement.

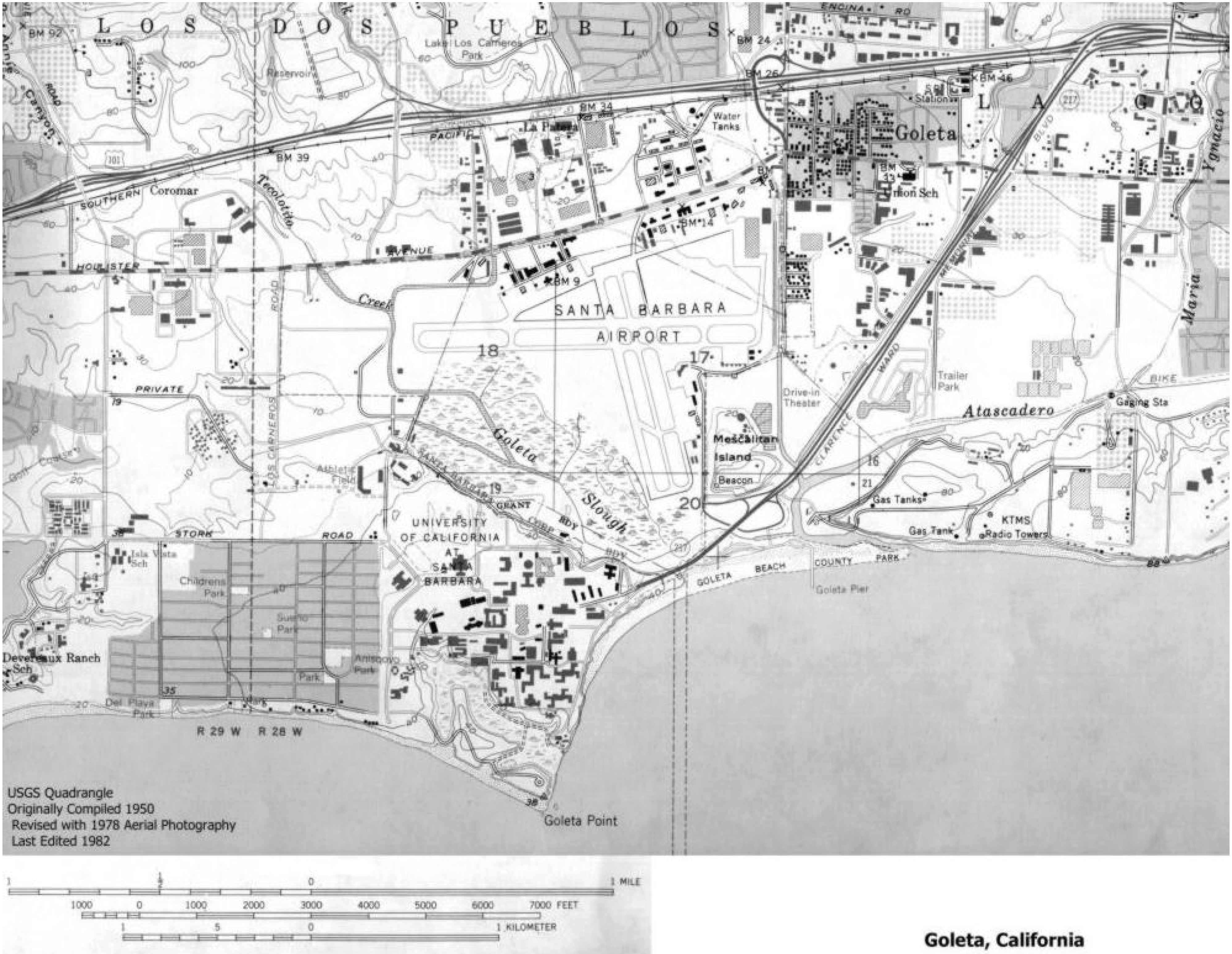
Relief Displacement is
A) Radially outward for features above the nadir elevation
B) Radially inward for features below the nadir elevation





B'
O
N'
Datum plane
A'
B


1954 (UCSB MIL)



Goleta, California

Representative Fraction (RF)	Word Statement
1:1200	100 feet to an inch
1:4800	400 feet to an inch
1:7920	⅛ mile to an inch (or 8 inches per mile)
1:9600	800 feet to an inch
1:15,840	¼ mile to an inch (or 4 inches per mile)
1:20,000	⅕ kilometer per centimeter (or 5 centimeters per kilometer)
1:24,000	2000 feet to an inch
1:25,000	¼ kilometer per centimeter (or 4 centimeters per kilometer)
1:31,680	½ mile to an inch (or 2 inches per mile)
1:50,000	½ kilometer per centimeter
1:62,500	1 mile to an inch (approx.)
1:63,360	1 mile to an inch
1:100,000	1 kilometer per centimeter
1:125,000	2 miles to an inch (approx.)
1:126,720	2 miles to an inch
1:250,000	4 miles to an inch (approx.)
1:253,440	4 miles to an inch
1:500,000	5 kilometers per centimeter
	8 miles to an inch (approx.)
1:506,880	8 miles to an inch
1:1,000,000	10 kilometers per centimeter
	16 miles to an inch (approx.)

1:1,000,000	10 kilometers per centimeter
	16 miles to an inch (approx.)
1:1,013,760	16 miles to an inch

Linear Measure

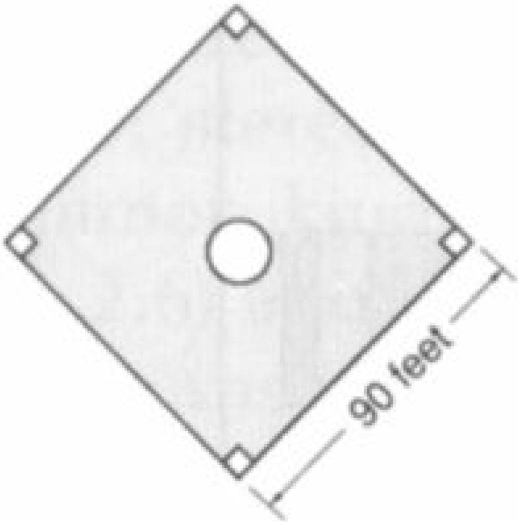
Given	Multiply by	To Yield
Nautical miles	1852.0	Meters
Statute miles	1609.3	Meters
Yards	0.9144	Meters
Feet	0.3048	Meters
Inches	0.0254	Meters
Meters	0.00054	Nautical miles
Meters	0.00062	Statute miles
Meters	1.094	Yards
Meters	3.281	Feet
Meters	39.37	Inches

Areal Measure

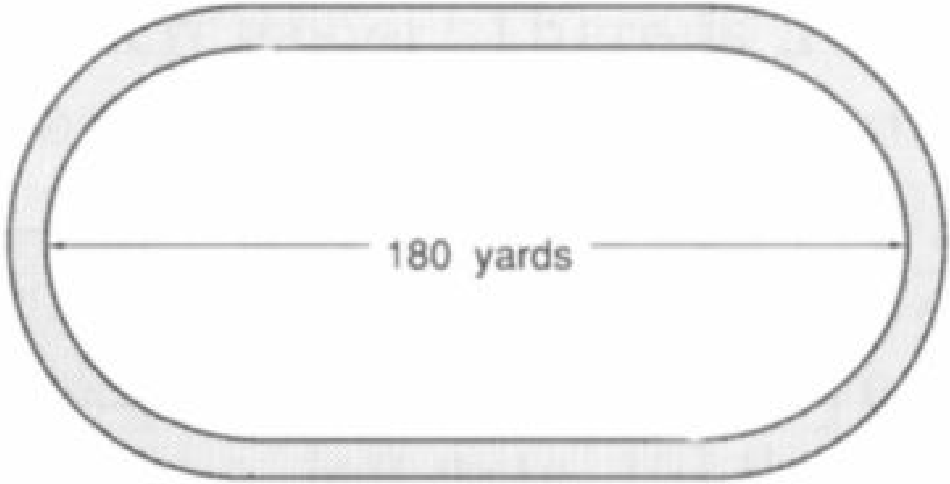
Given	Multiply by	To Yield
Sq. miles	2.59	Sq. kilometers
Acres	0.405	Hectares
Sq. yards	0.836	Sq. meters
Sq. feet	0.0929	Sq. meters
Sq. inches	6.452	Sq. centimeters
Sq. kilometers	0.386	Sq. miles
Hectares	2.471	Acres
Sq. meters	1.196	Sq. yards
Sq. meters	10.764	Sq. feet
Sq. centimeters	0.155	Sq. inches

Sq. meters	10.764	Sq. feet
Sq. centimeters	0.155	Sq. inches

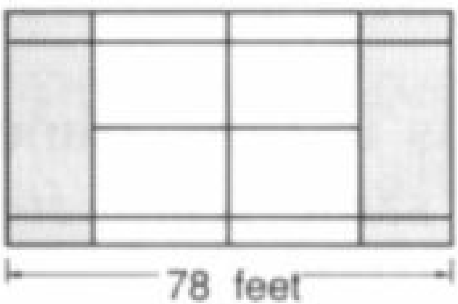
Baseball Diamond



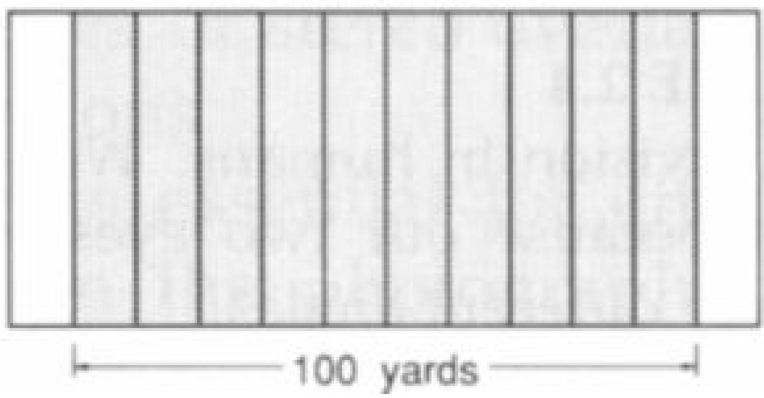
Standard 1/4 Mile Track



Tennis Court

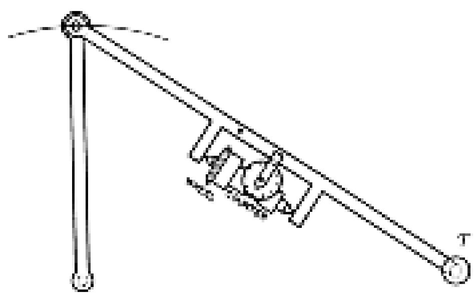


Football Field

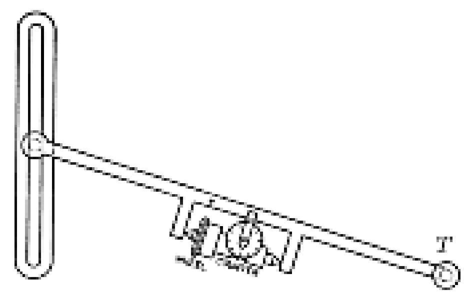


Planimeters

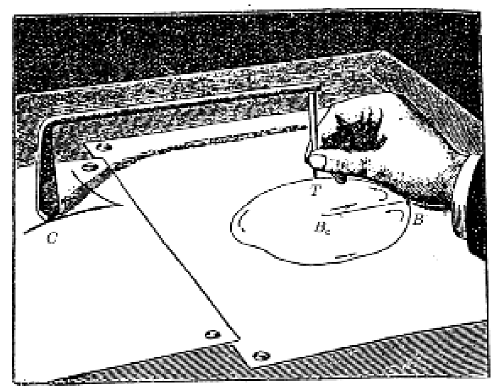
A planimeter is a mechanical instrument used to compute the area of a planar region. Here are some examples.



Polar Planimeter



Linear Planimeter



Prytz or "Hatchet" Planimeter

How planimeters work.
Using a planimeter to prove the isoperimetric inequality.

A Plethora of Planimeters!

Other Planimeter Web Sites

General

- How to Use A Planimeter
- Larry's Planimeter Planet
- University of Würzburg Mathematics Library (in German)

Historic Information & Photos of Antique Planimeters

- The Museum of HP Calculators -- Several good photos
- Antiques of Science & Technology

Mathematical Details

- The Incredible Inedible Planimeter
- The Computer Museum
- Paul Kunkel's Mathematics Lessons: The Planimeter
- Planimeters & Green's Theorem

Antique Dealers and Auctions That Often Have Planimeters

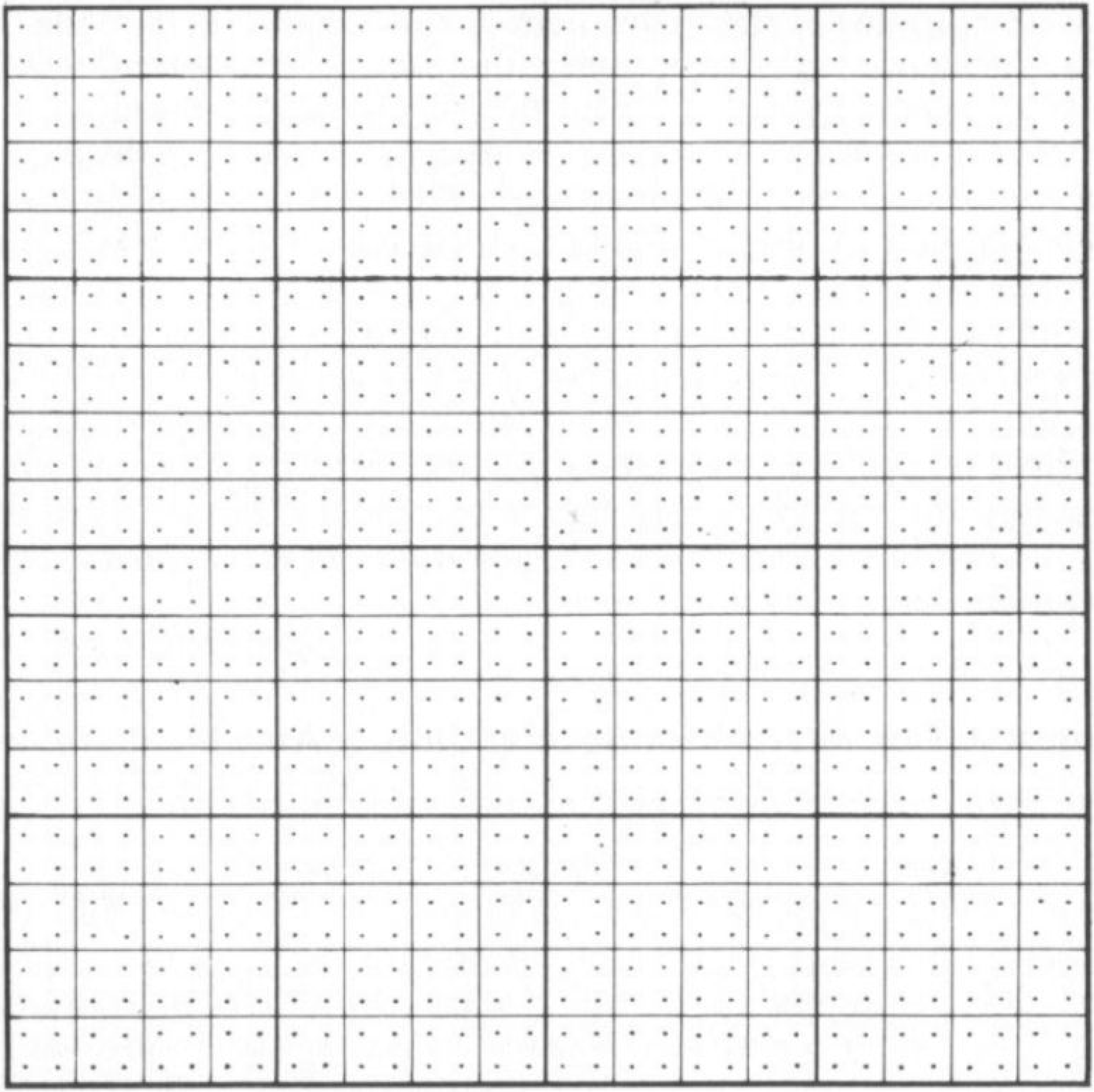
- The Gemmary Antique Scientific Instruments -- This is a great site with excellent photos
- eBay Search for "planimeter" or click on "Antiques" and then "Science Instruments."

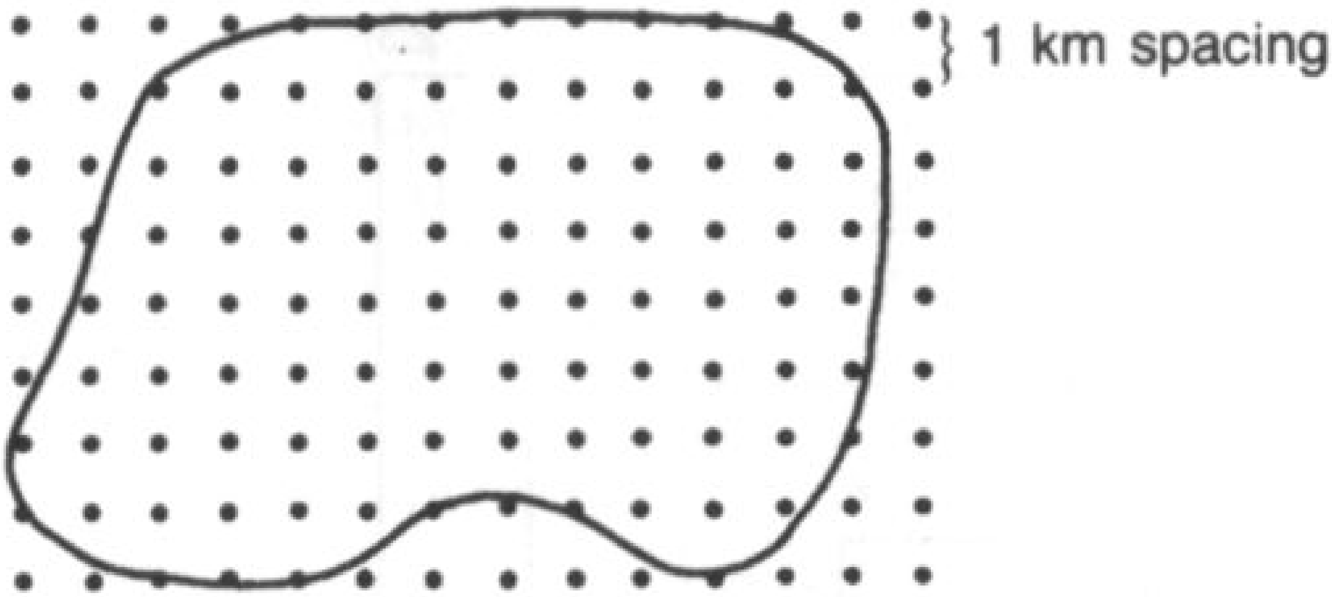
Companies Selling Modern Planimeters

- Lasico
- ASC Scientific

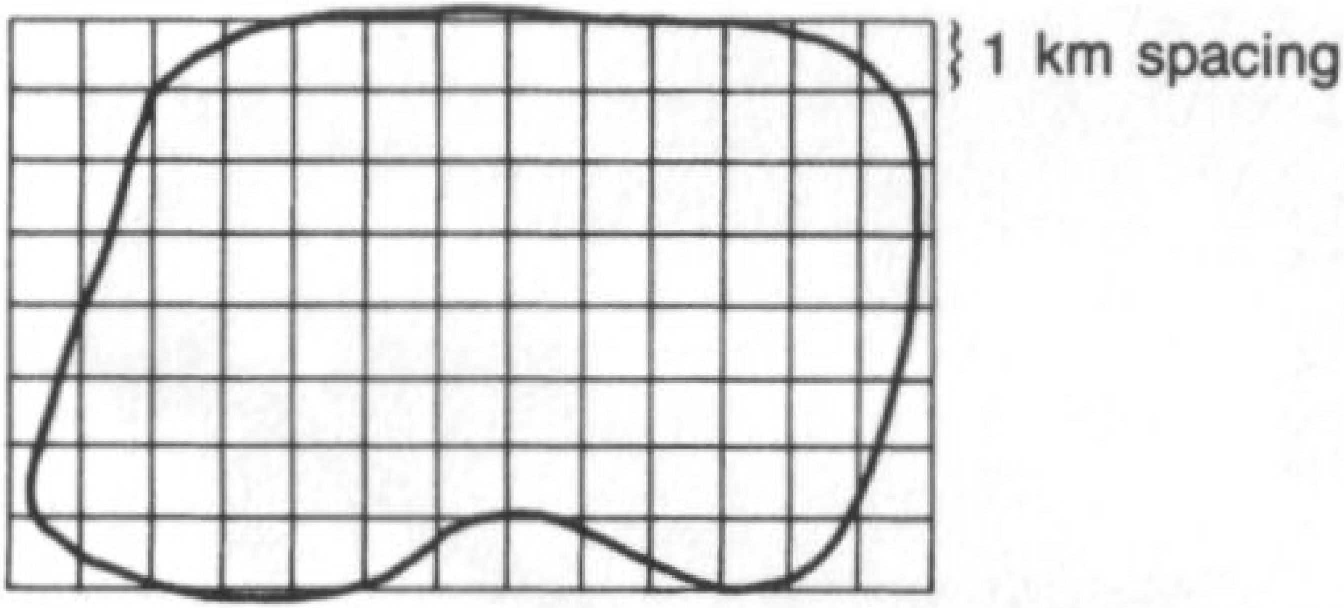
Search
Try using one of the web search engines to search for "planimeter." You will get lots of "hits," most of which will illustrate the variety of ways planimeters are used in science, technology, medicine, archeology, and many more areas.

Last update 4 February 2004
Robert Foote

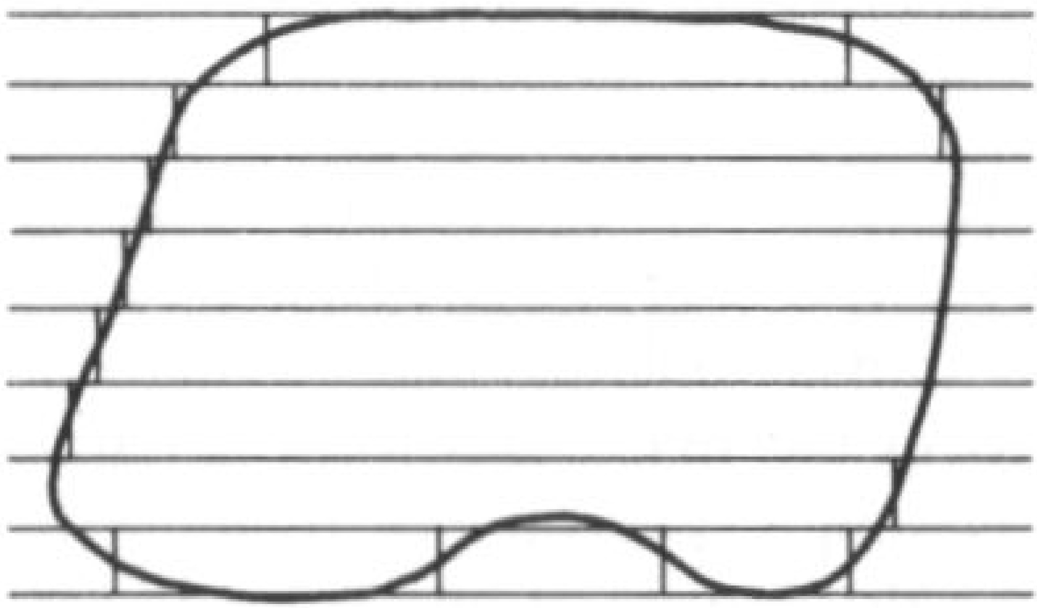




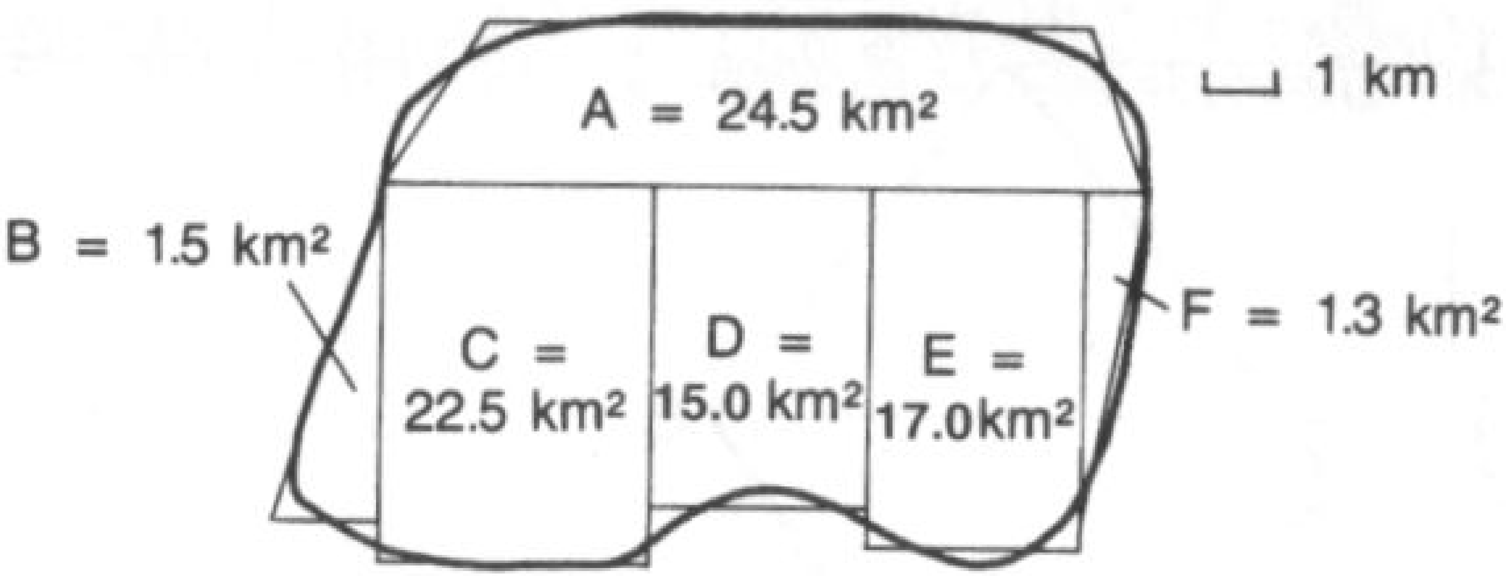
Area = [dots within + 1/2(dots on boundary)] × dot value
= [75 + 1/2(18)] × 1 km² = 84 km²



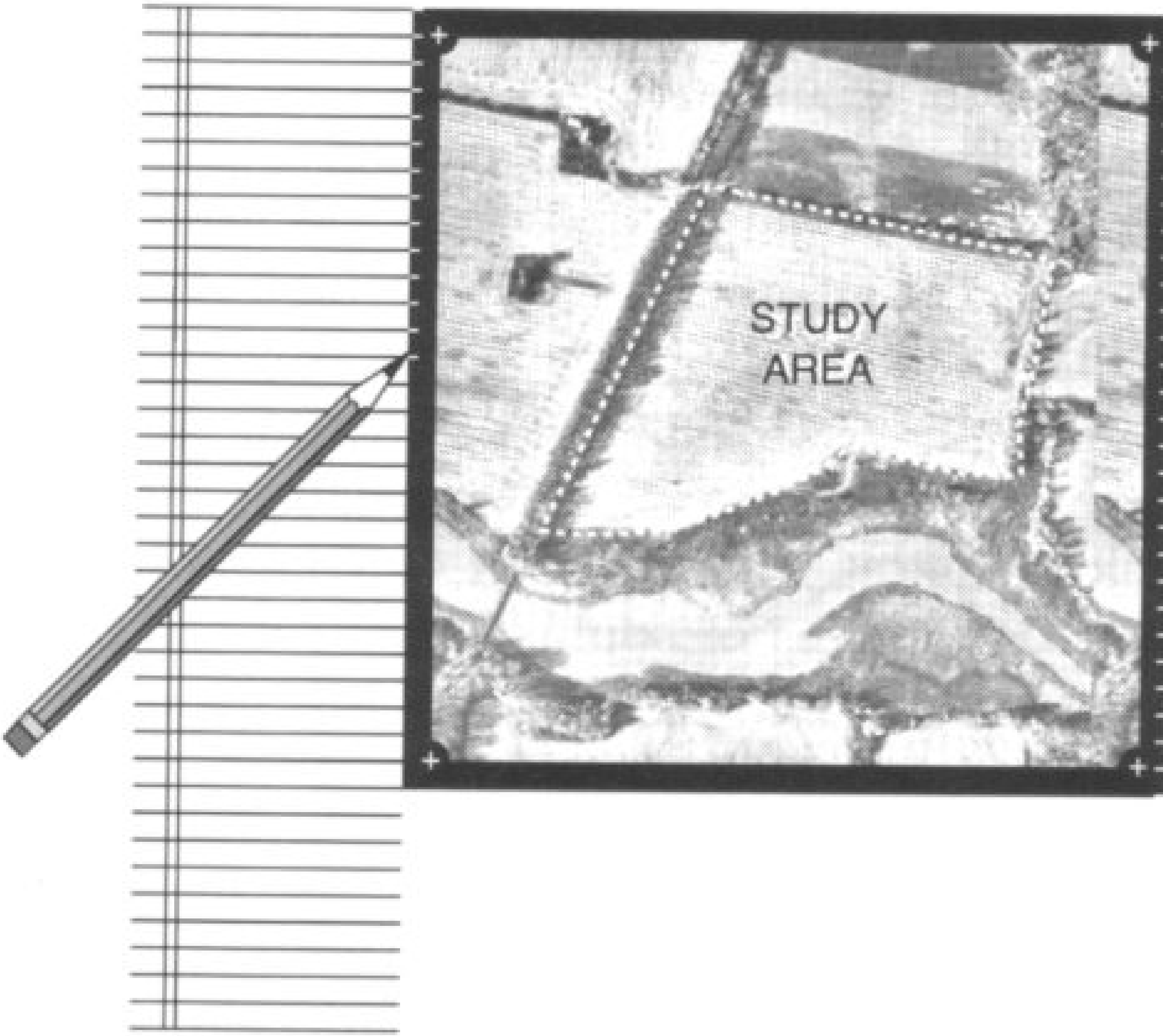
$$\text{Area} = [(\text{full cells}) + 1/2(\text{partial cells})] \times \text{cell value}$$
$$= [71 + 1/2(26)] \times 1\ \text{km}^2 = 84\ \text{km}^2$$

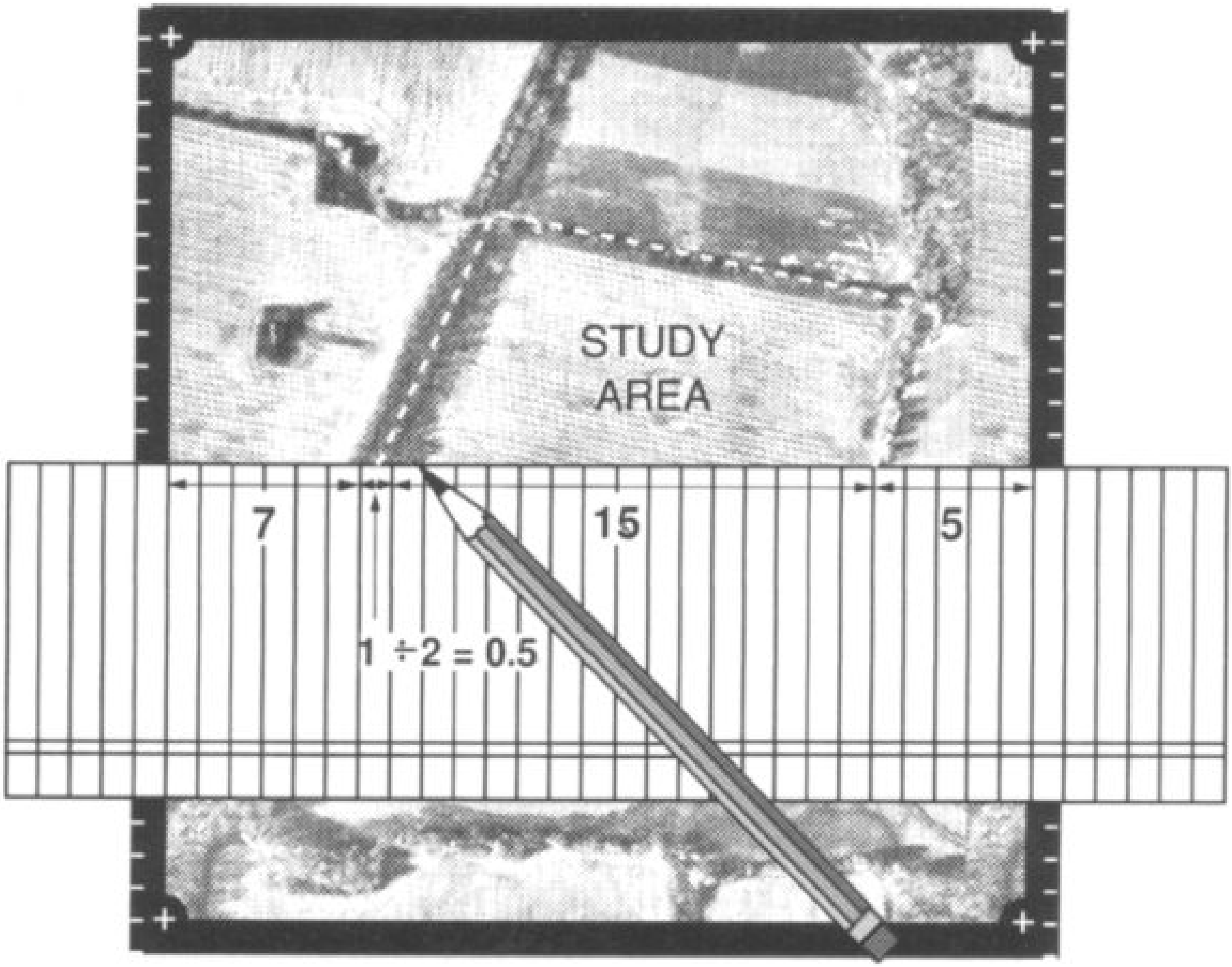


Area = total length of strips × width
= 84.2 km × 1 km = 84.2 km^2



$$\text{Total area} = \text{Areas }(A + B + C + D + E + F)$$
$$= 81.8 \text{ km}^2$$


STUDY
AREA


STUDY
AREA
7
15
5
1 ÷2 = 0.5